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Faustmann in the Sea: Optimal Rotation in Aquaculture

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Abstract *In this article an extended version of the well-known Faustmann model is developed for solving the rotation problem in fish farming. Two particularly important aspects of the problem are emphasized: First, the possibilities for cycles in relative price relationships and second, restrictions in release time for certain species. An illustration of the model based on assumptions from salmon farming shows that the inclusion of these two features has major influences on rotation time, and hence harvest weight.*

Key words Aquaculture, optimal rotation, dynamic programming, relative prices.

JEL Classification Code Q22.

Introduction

As fish farms become larger and the industry becomes more competitive, optimal production planning and efficient management practices become key factors for profitability. Among the most important managerial activities in production planning is that of determining the optimal rotation; *i.e.*, finding the best sequence of release and harvesting. This plan impacts the farm's cash flow as well as the allocation of limited production resources, such as feed, fish, space, and environmental resources (Cacho 1997).

The *rotation* problem in fish farming has, together with other fish farming management problems, a lot in common with problems already solved in forestry and animal husbandry.[1] Bjørndal (1990, p. 139) states: "Conceptually, aquaculture is more similar to forestry and animal husbandry than to traditional ocean fisheries," whereas Karp, Sadeh, and Griffin (1986) established the link between the rotation problem in fish farming and that of forestry. During the last decade several models for the *optimal harvesting* of farmed fish have been developed.[2] However, most of these studies consider only a one-shot decision, instead of treating the problem in a dynamic context focusing on decisions for optimal rotation. With space (volume) as a constraint, this is potentially a serious shortcoming of the traditional models. As the marginal biomass value decreases over time, harvesting makes room for new re-

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[1] Asche (2008) describes how the development of modern aquaculture shares many features of modern agriculture.
[2] Some articles related to the problem of optimal harvesting include: Leung (1986), Leung and Shang (1989), Hean (1994), Rizzo and Spagnolo (1996), and Forsberg (1999).

leases of younger and faster growing fish. I will argue that considering a one-time investment gives only a rough estimate, at best, of the optimal harvest time.

This article presents a dynamic programming model that solves the rotation problem in aquaculture. Two particularly important aspects of the optimal rotation problem will be emphasized. First, the possibility of releasing juvenile fish at any time of the year is limited for many species and can be an important constraint. Second, due to seasonalities in supply and demand, relative price relationships between different sizes of fish vary throughout the year. Hence, large fish would reach relatively higher prices than small fish at some times of the year, while the opposite might be the case at other times of the year (Asche and Guttormsen 2001). When solving for optimal harvesting time, the model should be able to include all appropriate relative price relationships.

The model can be used for different species of fish and different farming technologies. However, the phrasing and illustration of the model will be in terms of salmon farming. There are two major reasons for this. Salmon is one of the most successfully farmed fish, and salmon farming is a complex production process with features that illustrate important aspects of the models.

In what follows, the rotation problem in fish farming is outlined, the main characteristics of fish farming are stressed, and links to similar problems in other industries are described. Then, previous models from the literature are briefly reviewed before the new model is presented. Finally, the usefulness of the model is illustrated before the findings are summarized.

Optimal Rotation in Aquaculture

Farming techniques and practices vary between species as well as between firms farming the same species. Different technologies include ponds, raceways, pens, tanks, and cages. However, the basic principles are the same. Very simplified, the process of fish farming can be described as follows: the farmer releases certain amounts of recruits (juvenile fish) into pens or ponds, feeds them for some time, and harvests them when they have reached an appropriate market weight. When the fish are harvested, space becomes available for new juvenile fish. The farmer can then decide if he or she wants to market small fish by short rotations or larger fish with longer rotations. For some species it is possible to start a new generation at any time of the year, while starting new rotations are limited to certain times of the year for other species. The farmer's two most important decisions in the production process are then: (*i*) when to transfer the juvenile fish to the pen, and (*ii*) when to harvest the fish; *i.e.*, when to start and when to end a rotation.

The Faustmann solution has long been established as the correct approach to solving rotation problems.[3,4] The solution can be explained as follows. The tree should be cut at age *T* when the marginal increment to the value of the trees equals the sum of the opportunity cost of investment tied up in the standing trees and in the site (independent of whether this site should continue as forest or be converted, for example, into parking lots). The Faustmann model in it simplest form requires that a new rotation is started at the same time as the previous one ends. This is not realistic for several farmed species. Salmon smolts, for instance, can only be released during

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[3] The Faustmann article originally appeared in German, but was later translated to English (Faustmann 1849).

[4] In addition to the similarities between fish farming and forestry, rotation problems in aquaculture also share similarities with replacement problems in traditional livestock production. See Kennedy (1986).

certain periods of the year.[5] Other species are dependent on wild fingerlings or larvae; such is the case in most of the shrimp farming industry and other capture-based aquaculture. An optimal harvesting model for aquaculture should be able to take this constraint into account. Note that including restrictions on starting time means that there will be no universal optimal harvest weight. Instead, the optimal rotation will be different for different groups of fish based on when the rotation starts.

A problem related to prices that is apparent in fish farming, but not so relevant for forestry, is relative price relationships among sizes of fish.[6] While a tree in the forest will be of only a marginally larger size as time goes by, a growing salmon will "jump" from one quality class to another with certain distinct characteristics every time it develops into a new weight-class. Several studies indicate that the farmer receives different prices for different sizes. If the relationships between prices for different weight classes are constant, they can easily be incorporated into the Faustmann model. However Asche and Guttormsen (2001) examine relative prices (*i.e.*, relationships between prices for different weight classes) for salmon and find that relative prices vary throughout the year; *i.e.*, there exists patterns in the relative price relationships. For some part of the year large fish receive a higher price per kilo than small fish, and at other times of the year the situation is reversed. A harvesting model should be able to take these different deterministic price relationships into account. This aspect is also of especial interest for new farmed species that compete in the same markets as their wild cousins. Kristofersson and Rickertsen (2004) find that prices for different sizes of cod vary during the season. Such cycles might create opportunities for aquaculture, as fish farmers can take advantage of the seasonalities in the wild fisheries and market their fish out of phase with the wild products.

Previous Research on Optimal Harvesting in Aquaculture

While several studies exist on optimal harvesting problems for farmed aquatic species, I will argue that most of these studies do not address all the important aspects of the problem. In a chronological review of the period 1974 to 1996, Cacho (1997) finds that the most popular species for modeling are shrimp, prawn, and salmon. While some of the articles focus on specific species and technologies, others claim to be more general and applicable for different technologies and species. In the following, a selection of important studies will be briefly reviewed.

Karp, Sadeh, and Griffin (1986) consider the problem of determining optimal harvest as well as restocking time and level for farmed shrimp. They first consider the case where production occurs continuously, modeled as a deterministic, continuous-time autonomous control problem. Harvest and subsequent restocking are modeled as "jumps" in the biomass. Their contribution to the traditional Faustmann solution is that the optimality conditions determine the restocking level as well as the harvest level. Second, they consider the situation where the environment is uncontrolled, modeled as a stochastic control problem. They then proceed to solve it with dynamic programming. However, their model is not flexible enough to include

[5] Due to biological and economic reasons, smolts can only be transferred to sea during a certain period of the year (March-October). In nature, salmon spawn during late spring and hatch normally in January. Therefore, most salmon produced "are born" in January. The supply of smolts is consequently limited in other periods. Smolts do not tolerate cold weather well, so release during the winter months is connected with great risk of loss.

[6] For most species, the price per kg will increase with the size of the fish (Asche, Guttormsen, and Tveterås (2001).

different relative price relationships, and they assume that a new rotation can start at any time of the year.

Bjørndal's (1988, 1990) main point is that fish in a pen are nothing else than a particular form of growing capital. Hence the objective of finding the optimal harvesting times is similar to maximizing the present value of an investment. Bjørndal presents a model in which he illustrates the changes in biomass value over time as a function of growth, natural mortality, and fish prices. He then adds costs to the model and presents a comparative statics analysis of the effects of changes in the parameters on optimal harvest date. However, the model is in terms of a one-time investment and what happens after the harvest is not considered. Bjørndal (1988, p. 153) admits that: "It is not sufficient to merely consider a single harvesting time. The problem in question represents an infinite series of investments rather than a one time investment." While he briefly presents a Faustmann-like solution to the problem, the model can neither treat the problem containing restrictions on release time nor treat dynamics in relative price relationships.

Several authors have extended Bjørndal's model to emphasize specific aspects of the problem. Arnason (1992) introduces dynamic behavior and presents a general comparative dynamic analysis. He also introduces feeding as a decision variable. Heaps (1993) deals with density-independent growth, whereas Heaps (1995) allows for density-dependent growth and also looks at the culling of farmed fish. Mistiaen and Strand (1998) demonstrate general solutions for optimal feeding schedules and harvesting time under conditions of piecewise-continuous, weight-dependent prices. None of these studies consider the rotation problem.

As this brief review illustrates, only Karp, Sadeh, and Griffin (1986) and Bjørndal (1988, 1990) discuss the rotation problem. However both assume that when one year-class is harvested, the next one is released immediately. This again implies that recruits are available throughout the year, which is not the case for a number of important species (salmon, among others). None of the articles discuss the problems of dynamics in relative price relationships. This is a serious weakness of the model because changes in relative prices are significant for some aquaculture species (Asche and Guttormsen 2001).

An Extended Faustmann Model

The Faustmann model, developed in 1849, has been established as a benchmark model for determining optimal timber rotation age. Faustmann showed that the value of a forest can be expressed as a sum of net cash flow over an infinite time period and that a forest owner's goal is to choose rotation so that the value of a forest is maximized. Translated to aquaculture, Faustmann's rule says that it is optimal to harvest a cohort at time T^*, when the marginal increment to the value of the cohort equals the sum of the opportunity cost of investment tied up in the cohort and in the empty pen; *i.e.*, when marginal *benefits* from delaying harvest are equal to marginal *cost* of delaying harvest. Marginal costs of delaying harvests include not only foregone interest payments, but also the value lost from delaying the next rotations.

To take into account the problematic assumptions about the possibilities of continuous release of juvenile fish and constant relative prices, I will now extend the Faustmann model. For pedagogical reasons I have divided the model into two parts, one part for the harvesting problem and one for the release problem; *i.e.*, when there are fish in the pen and when the pen is empty. With fish in the pen the farmer can, at every decision stage, either harvest the fish or wait. If he harvests he will have an empty pen in which he can either start a new rotation or not. Even though the model is phrased in two parts, it is solved as *one* optimization problem.

I will present the model as a discrete dynamic programming model, were the objective is to maximize the value of the pen over an infinite time horizon. I start by defining the biological part of the model, and then add prices and cost to construct the full bioeconomic model.

Biomass is dependent upon the number of fish and the weight of each fish. I assume homogeneity among the fish (*i.e.*, all fish grow at the same speed, so that I can speak of a representative fish). The biomass of a year class at time t, b_t, is then calculated as:

$$b_t = n_t w_t,$$

where n_t is the number of fish and w_t is the weight of the representative fish. As time increases, two processes will influence the growth of biomass; some fish will die, and the others will gain weight. The number of fish at every stage is consequently a function of number of fish released, n_0, and mortality, m. Number of fish at time $t + 1$, n_{t+1}, will consequently be: $n_{t+1} = (1 - m)n_t$. Mortality, m, can be treated as constant or varying throughout the year as a function of the size of the fish and the time of year. Those fish that do not die will grow and gain weight. I assume that the fish grow according to some well-defined growth function, $g(w_t, temp)$, which is a function of initial weight, w_t, and water temperature.[7] For simplicity, I assume that water temperature follows some site-specific temperature table such that *temp* can be substituted with month number, k. Weight of each individual fish at time $t + 1$ will then be: $w_{t+1} = g(w_t, k)w_t$. Hence biomass will be $b_t = w_t n_t$ and $b_{t+1} = w_{t+1}n_{t+1}$ in periods t and $t + 1$, respectively, where $w_{t+1} = g(w_t, k)w_t$ and $n_{t+1} = (1 - m)n_t$.

I have then established the biological relationships and can add input and output prices. The value of the standing biomass at time t, V_t, is found by multiplying price times biomass. Defining price per kilo as a function of weight, w, and season, presented by month number, k, I will have a price function $p_t(w_t,k_t)$. This function provides flexibility to the model, and I can model all sorts of relative price relationships. I will exemplify this price function in our example. This function is the same that is used in Forsberg and Guttormsen (2006). The value of the biomass at all t is hence $V_t = p_t(w_t, k_t)w_t n_t$. Given that the farmer decides to harvest the cohort, the value of the pen will be this value, plus the discounted value of future profit.

The objective of decision making for the fish farmer is to maximize the present value of net income streams to infinity. I assume first that the only decision variable is whether to harvest or to wait. If the farmer decides to wait, the fish will grow, and there will be some cost associated with the decision. However, to simplify I only include feeding cost and neglect harvest cost.

If the farmer chooses to delay harvest, the value of the pen will be the discounted value for the next period minus the incurred feeding cost; *i.e.* $- c^f + \alpha V_{t+1}$, where c^f are the feeding costs and α is the discount factor. I assume a constant feed conversion rate, which means feeding cost is a function of weight gain, and feed-price, *fp; i.e.*, the cost of feeding a cohort from t to $t + 1$,$c^f_{t\to t+1} = FCR(n_{t+1}w_{t+1} - n_t w_t)fp$. By adding it all together, I can now write the farmers maximization problem as follows:

$$V_t(w_t, n_t, k_t) = \max_{d_t}\left\{p_t(w, k)w_t n_t d_t - c^f_{t\to t+1} + \alpha V_{t+1}(w_{t+1}, n_{t+1}, k_{t+1})\right\}, \qquad (1)$$

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[7] Growth functions in practical fish farming are usually tabulated; *i.e.*, the table indicates how much a fish of size w will grow in one day with different water temperatures.

where:

$$d_t = \begin{cases} 0 - \text{wait} \\ 1 - \text{harvest} \end{cases}$$

$$w_{t+1} = g(w_t, k)w_t(1 - d_t)$$

$$n_{t+1} = (1 - m)n_t$$

$$c^f_{t \to t+1} = FCR(n_{t+1}w_{t+1} - n_t w_t)fp,$$

where d_t is the decision variable, taking 0 for wait and 1 for harvest, and all other variables are as presented above.

Immediately after harvest the pen is empty and the farmer can decide whether to release new fish or wait. Hence, he must decide at every stage whether releasing juvenile fish immediately, or waiting, maximizes the net present value of the pen. The value when releasing fish will be the discounted future value of the pen minus the cost of releasing, $-c_r$. With a decision to release, the weight of the representative fish the next period will be $(1 - \delta_k)w_t$, where δ_k is then a first day death rate. This parameter is the key for handling the problems of restrictions in release time. In periods where it is impossible to release juvenile fish, I set δ_k equal to one. In periods where it is possible to release but with high death rate, I can set δ_k equal to 0.5 *etc*.[8] The release decision is defined by the following:

$$V_t(0, k_t) = \max_{s_t}\left\{-c_r s_t + \alpha V_{t+1}(w_{t+1}, n_{t+1}, k_{t+1})\right\} \tag{2}$$

where:

$$s_t = \begin{cases} 0 \text{ not release} \\ 1 \text{ release} \end{cases}$$

$$w_{t+1} = (1 - \delta_t)s_t w_t.$$

By adding equations (1) and (2), I have a model that can solve the harvesting problem independent of restrictions in release time and with many relative price relationships. It is also relatively easy to add different costs; *i.e.*, release cost, harvest cost, insurance cost, *etc*., and then examine what happens with rotation time when changes occur in one or several of the parameters.

The model will then end up as a generalized version of the Faustmann problem. With growth independent of release time; *i.e.*, δ_k equals to zero (which means that it is possible to release fish during the whole year) and no seasonalities or fluctuations in prices or costs, the model will collapse down to the traditional Faustmann solution.

[8] For simplicity, I have assumed weight gain to be zero in the first period. This assumption is reasonable because the fish need some time in the pen to adapt to the new environment.

An Empirical Illustration

The model is relatively easy and flexible to use, and I will only illustrate the importance of extending the standard Faustmann model with some fairly simple numerical illustrations. The model is programmed in MATLAB from MathWorks Inc. and solved with a toolbox developed by Miranda and Fackler (2001). To emphasize the importance of extending the model, unnecessary details are ignored. The model is applied to salmon farming. This is based on the fact that salmon is one of the most successfully farmed species, and that salmon farming exhibits features that illustrate important aspects of the model.

Biomass is defined as the number of fish times fish weight, and for weight gains, a slightly updated version of the growth function provided in Bjørndal (1990) is used.[9] The growth function is defined as follows:

$$w(y) = 2.8y^2 - 0.7y^3, \tag{3}$$

where y is number of years from release. The function is tabulated for months. I have further set the monthly mortality rate to 0.8% and the interest rate to 7%. There is no need to define a specific n_0, since it is the relative change that is important. To simplify, release and harvest cost is set to zero, and feed cost is assumed to be proportional to growth. That means that the fish need the same amount of feed to grow regardless of whether they grow from one to two kilos or from four to five kilos. The models are then programmed and solved with different assumptions about release time and relative price relationship.

First, I solved the model with the standard Faustmann assumptions; *i.e.*, I assumed constant relative prices and the possibility to release smolts to sea through the whole year. This means that δ_k is equal to zero in all months. With a price per kilogram of all sized salmon of NOK 26, I find that optimal harvest time is after 21 months in sea when the fish have reached 4.8 kilos market weight. This is the Faustmann rotation length, indicating that at this time, the potential interest gained on the harvested value plus the growth in value for the new release is larger than the growth in value for the swimming biomass. This is shown in the second column in Table 1 under the heading 'Faustmann.' Given these assumptions, the harvest weight is also independent of which period the fish is released.

Next, I loosen the assumption about continuous release possibilities and restrict release time to seasons in the year were it is possible to start a salmon rotation. I then set δ_k to zero in March, April, May, August, September, and October and let δ_k equals one for the other months. All other assumptions are the same as in the first example. The optimal harvest weight is given in the third column in table 1 under the heading 'Constant Price.' As one can see, these restrictions give different harvest times depending upon starting time. For some starting times, harvest is postponed, while for other starting times, harvest is pushed forward. This, of course, makes sense. Following the Faustmann solution, fish released in March should be harvested in December. As such, the farmer would have an empty pen until March, and consequently three months without any production. It can be seen that only the fish released in August will be harvested at 4.8 kilos, and fish released at other times of the year will be harvested between months 19 and 23 (4.2–5.3 kilos).

To illustrate another of the advantages of the extended model, I also include a

[9] Bjørndal's (1990) growth functions are based on data from salmon farmed in 1988. Since then selective breeding, feed, and feeding technology have improved the farming such that the fish grow much faster.

Table 1
Optimal Harvest Weight and Time with and without Relative Price Relationship

	Harvest Weight in Kilos/Age (months)		
Release Time	Faustmann	Constant Price	Relative Price Relationship
March	4.82 (21)	4.24 (19)	3.63 (17)
April	4.82 (21)	5.36 (23)	3.32 (16)
May	4.82 (21)	5.10 (22)	6.47 (29)
August	4.82 (21)	4.82 (21)	6.03 (26)
September	4.82 (21)	4.54 (20)	5.60 (24)
October	4.82 (21)	5.10 (22)	5.36 (23)

non-constant relative price relationship. This relative price relationship is based on the results in Asche and Guttormsen (2001) as follows. Based on price observations for salmon of different sizes from 1993 to 2002, a monthly relative price index is constructed for each weight class. Three to five kilos is used as a base weight; *i.e.,* the price equals one. The price index for the other weight classes, 5–6 kilos for instance in January, is then calculated as:

$$\sum_{i=1993}^{n=2002} (p_{\mathrm{i,january}}^{5-6\mathrm{kilos}} / p_{\mathrm{i,january}}^{3-5\mathrm{kilos}}). \tag{4}$$

A graph of the price indexes is provided in figure 1. As can be seen, larger fish will usually be more valuable than smaller fish. However, this changes during the year. Looking at the optimal harvest results in the final column of table 1, one can see that the numbers change significantly. Fish released in April should now be harvested at 3.32 kilos after 16 months in the sea instead of at 5.36 kilos after 23 months in the sea. Also, for all other release times, the inclusion of the relative price relationship substantially changes the harvest time.

Concluding Remarks

The rotation problem in fish farming shares many features with rotation problems in forestry and traditional terrestrial livestock production. However, fish farming also exhibits specific features that demand a more flexible model than those constructed for other industries. In this article, such a model is presented. This model is general and flexible enough to treat different species and technologies. Two specific features for aquaculture are stressed: restrictions on release times and dynamic relative price relationships. To illustrate the strength of the model and the importance of extending traditional models, a simple example of the use of the model is presented. The empirical illustration shows the importance of a model that can treat different relative price relationships as well as restrictions on when it is possible to start a new rotation.

Asche and Guttormsen (2001) claim that: "we in general cannot say anything about the direction of the changes in the harvest time due to the cycles in relative prices." This claim is confirmed with the extended model since fish released at dis-

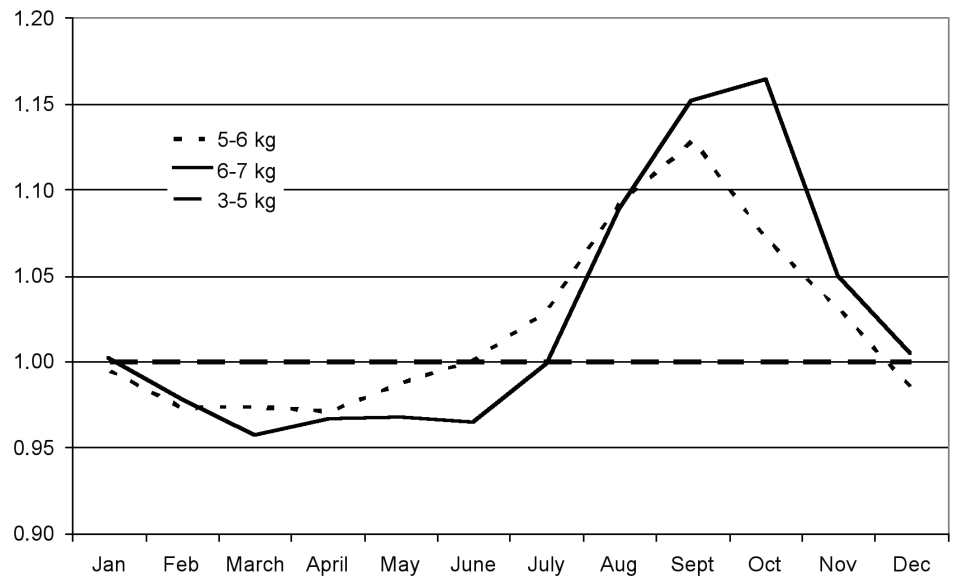

Figure 1. Price Index, Relative Price Relationship

Note: Based on historical observation 1993–2002.

tinct times of the year will have higher harvest weights when relative price relationships are included, while fish released at other times of the year will have lower harvest weights.

As fish farm enterprises become larger and the industry becomes more competitive, the timing of harvesting and marketing become key factors for success. The production plan has an impact on the cash flow from the farm as well as on the allocation of limited resources in production, such as feed, fish, space, and environmental resources. Consequently, a well-developed production plan can mean the difference between loss and profit for a fish farm.

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